THE MATHEMATICAL ASSOCIATION OF VICTORIA

MATHS TREATS   BY LUCIANA THE POSSUM

RECREATIONAL MATHS

Mathematics is not just about learning the curriculum, and doing exercises and tests. Many mathematical developments have come from people exploring and having fun. In fact, playing with numbers and finding patterns, or lack of them, can be a fruitful way of practising number skills and developing rules which describe patterns.

HAPPY NUMBERS

Take any positive integer and sum the square of its digits. Repeat this process until a pattern forms. If this process ends in the number 1 then the starting number is a happy number.

For example, 19 is happy, as the associated sequence is:

$1^2 + 9^2 = 1 + 81 = 82$

$8^2 + 2^2 = 64 + 4 = 68$

$6^2 + 8^2 = 36 + 64 = 100$

$1^2 + 0^2 + 0^2 = 1$

Once the sequence reaches 1, it will stay there. Can you see why? If a sequence does not reach 1, then the starting number is an unhappy number.

ACTIVITY

What is the smallest happy number? Is there a largest happy number or do they go on forever? How many happy numbers are there up to 100? Will rearranging the digits of a number affect its happiness? Consider 326 and 362. What happens if you add (or remove) a zero anywhere within the number?

All happy numbers follow a sequence that eventually ends in 1. What sort of sequences do unhappy numbers follow?

What happens if, instead of squaring, you cube the digits? These are the kind of fun questions mathematicians like to ask.

PERFECT NUMBERS
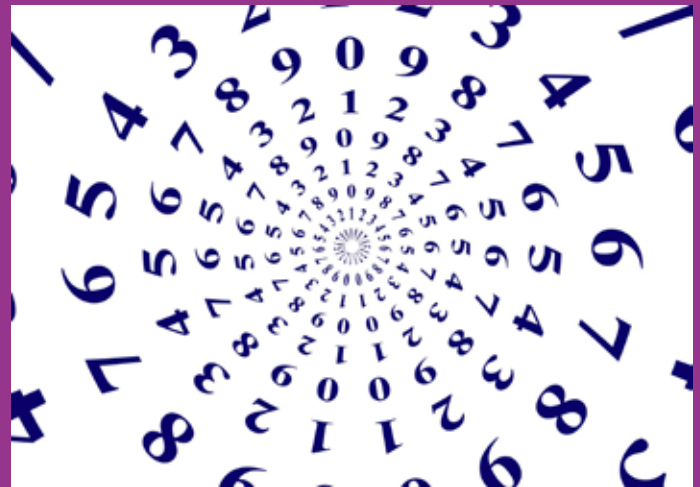
A positive integer has at least two factors, 1 and itself. A perfect number is a positive integer that is equal to the sum of all its factors except the number itself. The first perfect number is 6 which has the factors 1, 2, 3, and 6, where $1 + 2 + 3 = 6$. A definition of perfect numbers was found in Euclid's famous book, *Elements*, and the first four perfect numbers have been known for 2000 years. The 48th perfect number was identified in 2013 and comprises 34 850 340 digits. There is at least one perfect number which is also a happy number. Can you find it?

ACTIVITY

Use Excel or write code to explore happy numbers and perfect numbers. Can you find the next three perfect numbers? What is the relationship between a perfect number and the sum of all its factors? Are the perfect numbers you have found odd, even, or a mixture? Make up your own special numbers. Think of a fun name and then make up a rule for them.

REFERENCES AND FURTHER READING

RECREATIONAL MATHEMATICS
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IMAGES
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Other images from Pixabay